Exercise 38

Water temperature affects the growth rate of brook trout. The table shows the amount of weight gained by brook trout after 24 days in various water temperatures.

Temperature (°C)	15.5	17.7	20.0	22.4	24.4
Weight gained (g)	37.2	31.0	19.8	9.7	−9.8

If $W(x)$ is the weight gain at temperature x, construct a table of estimated values for W' and sketch its graph. What are the units for $W'(x)$?

Source: Adapted from J. Chadwick Jr., "Temperature Effects on Growth and Stress Physiology of Brook Trout: Implications for Climate Change Impacts on an Iconic Cold-Water Fish." Masters Theses. Paper 897. 2012. scholarworks.umass.edu/theses/897..

Solution

$W'(x)$ is the rate at which the weight is increasing with respect to temperature (units of °C/g). To obtain the values of $W'(x)$, calculate the slope of the secant line going through two adjacent x values. At $x = 15.5$, for example,

$$W'(x) = \frac{W(17.7) - W(15.5)}{17.7 - 15.5} = \frac{31.0 - 37.2}{2.2} = -2.82.$$

At $x = 17.7$, there are two secant lines.

$$W'(x) = \frac{W(17.7) - W(15.5)}{17.7 - 15.5} = \frac{31.0 - 37.2}{2} = -2.82$$

$$W'(x) = \frac{W(20.0) - W(17.7)}{20.0 - 17.7} = \frac{19.8 - 31.0}{2.3} = -4.87$$
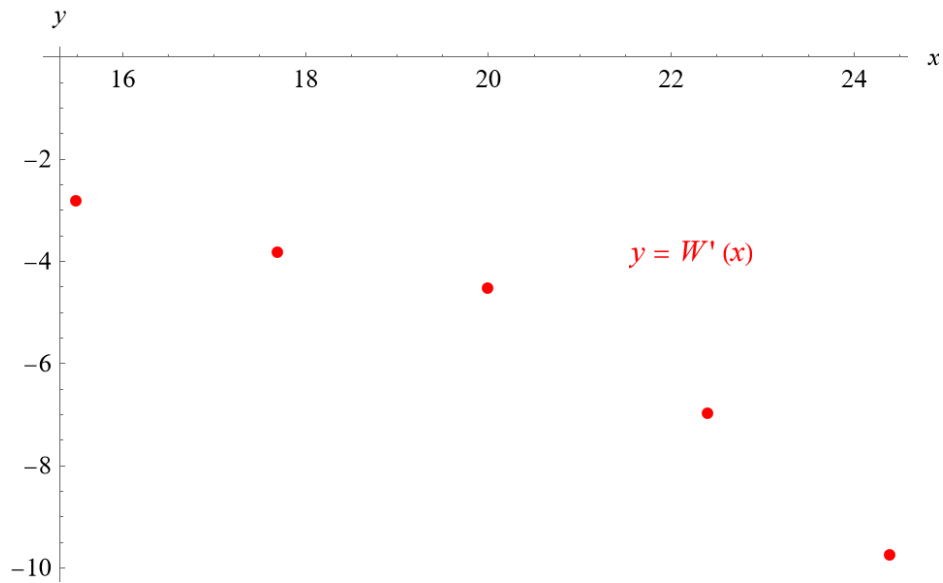
At such times where there are two possible secant lines, take the average for the best estimate.

$$\frac{(-2.82) + (-4.87)}{2} \approx -3.84$$

Below is a table of estimated values for $W'(x)$.

x	$W(x)$	$W'(x)$
15.5	37.2	−2.82
17.7	31.0	−3.84
20.0	19.8	−4.54
22.4	9.7	−6.98
24.4	−9.8	−9.75

Below is a graph of W' versus x.

$y = W'(x)$

It shows that with increasing temperatures, the brook trout grow less and less properly.